Report 8

16. Magnetic cannon

Consider a line of steel balls that are stuck to a strong magnet. If an additional ball collides with the line, the final ball will be ejected at high speed. Determine the maximum speed that the final ball can have. How does this speed depend upon the position of the magnet in the line and other properties of the system?

Introduction

First, let us estimate the origin of the phenomenon observed. The high-speed departure of the last ball can be explained as follows: the first ball is affected by attractive force of magnetic field when it approaches the magnet, so the ball accelerates and obtains a high impulse. This impulse is transferred to the other balls in the line when the first ball strikes the magnet (similarly to the Newton's cradle). It should be noticed that the last ball is located quite far from the magnet and the impact of magnetic field on it's motion is weak. However, the energy dissipates when the impulse is transferred from ball to ball, so the last ball obtains only a part of the first ball's impulse.

We have stated the following objectives to solve this problem: to build the theoretic model of the process, to estimate the optimal experimental conditions empirically and to conduct some experiments, as well as to verify correspondence between experimental results and theoretical data.

The theoretical model

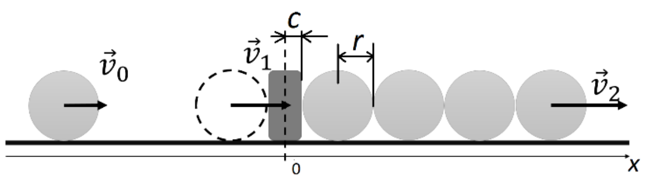

Figure 1. The scheme of the process.

The energy conservation law-based approach was chosen as the basis of our theoretical model.

Kinetic energy of the striking ball (E_{k1}) is a sum of its initial kinetic energy (E_{k0}) and work of magnetic field applied to move the ball (A_1).

$$E_{k1} = E_{k0} + A_1 \tag{1}$$

Knowing attractive force that affects the ball in magnetic field (F_1), it's possible to express A_1 as follows:

$$A_1 = \int_{-\infty}^{-r-c} F_1(x)dx \tag{2}$$

Kinetic energy of the striking ball (E_{ki}) is a sum of its translational and rotational energy (due to rotation around the instantaneous ball-centered axis).

$$E_{k1} = \frac{mv_1^2}{2} + \frac{I\omega^2}{2} \tag{3}$$

Substituting equations for $\omega = \frac{v}{r}$ and the ball's moment of inertia $I = \frac{2}{5}mr^2$ in (3) it can be expressed that

$$\frac{mv_1^2}{2} = \frac{E_{k0}+A_1}{1.4} \tag{4}$$

It should be noted that only the translational energy is transmitted due to collision because the rotational energy turns into zero. Thus, total post-collisional energy of the system (E_2) is equal to translational energy of the striking ball. This energy turns into kinetic energy of the ball that departures the system (E_{k2}). But work of magnetic field during the ball's motion (A_2) and energy loss (ΔE) should also be considered:

$$E_2 = E_{k2} + A_2 + \Delta E \tag{5}$$

The work of the field can be expressed in a similar way. But this work will be negative because force of the magnetic field that affects the ball (F_2) tends to reduce the ball's speed.

$$A_2 = \int_{2r\left(n-\frac{1}{2}\right)+c}^{+\infty} F_2(x)dx \tag{6}$$

Let us note that $F_1(x) \neq F_2(x)$ because the balls to after the magnet distort magnetic field substantially. Thus, A_1 and A_2 depend on number of balls (n).

The result of (4) substitution into (5) is:

$$E_{k2} = \frac{E_{k0}+A_1(n)}{1.4} - A_2(n) - \Delta E(n) \tag{7}$$

Knowing E_{k2} makes it possible to estimate the speed of the ejected ball. It should be mentioned that some characteristics that affect the speed of the ball cannot be evaluated theoretically and should be found empirically.

Experimental setup to measure the speed

Let's introduce some qualitative reasons that defined our choice of the experimental setup. It should be noticed that we used the balls from ball-bearing for all experiments.

First, the direction of the striking ball should be precisely defined to ensure centrality of the strike. For that reason, we placed the magnet and the balls in a duct. The magnet was fixed to reduce dissipation in the system and avoid displacement of the strike due to its motion.

The rectangular magnets were used to facilitate the experiment.

The first ball was placed at the distance where the initial speed is zero, i.e. the ball started moving only due to the magnetic field.

As a result, the following experimental setup has been constructed (Figure 2):

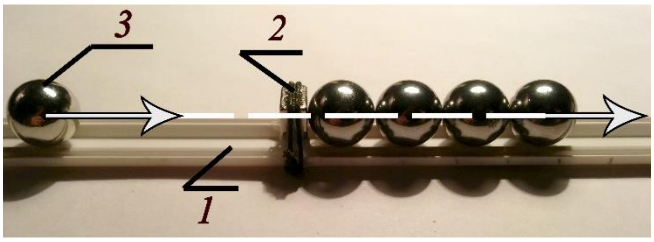

Figure 2. The experimental setup. 1 – a duct; 2 – a magnet; 3 – balls.

Departure of the last ball was filmed using a high-speed camera (1000 fps), and its speed was measured using the storyboard.

Diameter of the ball and the number of balls after the magnet were changed during the experiment to figure out conditions resulting in the maximal speed. It's important to note that the ball hasn't been rotating about 3-4 cm right after it had been dashed out. The speed was measured exactly on this distance.

The experimental setup to measure the force of magnetic field, affecting the ball

The forces of pulling the ball off the magnet in different places of the investigated chain are essential to describe the process and substitute the obtained functions ($F_1(x,n)$ and $F_2(x,n)$) into the quantitative calculations - (2) and (6).

The $F_2(x,n)$ dependence has been investigated using the paper spacers that were placed between the ball being pulled off and the other part of the system. The magnet and the whole system were firmly fixed. The electronic dynamometer has been used as a measuring instrument. Figure 3 shows an example of a measuring setup $F_2(x,3)$.

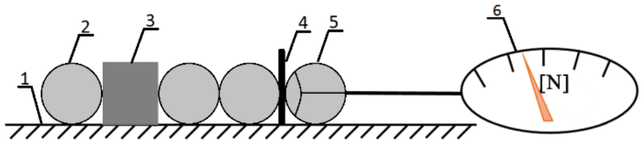

Figure 3. The experimental setup for measurement of pull-off forces. 1 – base; 2 – striking ball; 3 – magnet; 4 – paper spacers; 5 – ball being pulled off; 6 – dynamometer.

Based on physical meaning, the approximating function used was $\sim\frac{1}{x^2}$. It allowed us to describe the experimental results precisely enough.

As a result, $F_1(x,n)$ and $F_2(x,n)$ functions have been obtained for all the systems under study.

Experimental evaluation of the recovery factor after the strike

Recovery factor α is a unitless characteristic that shows what part of energy conserves while being transferred from one ball to another. The magnet's absorption of the strike energy is deemed as equal to the ball's one.

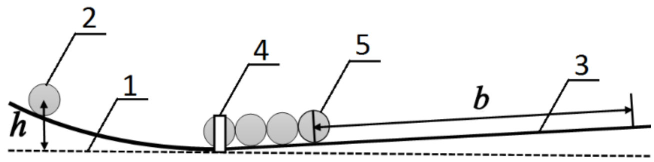

Figure 4. Measuring of the recovery factor α. 1 – base; 2 – striking ball; 3 – duct; 4 – fixed ball; 5 – ball being pulled off.

Potential energy of the ball to the left is turned into its kinetic energy during the experiment. It strikes the system of balls, transmitting impulse to the last ball that is dashed out. The first (stricken) ball in the chain was fixed to guarantee fixedness of the balls. It's important to note that incidence angles of plains were small enough to ensure centrality of the strike.

Elevation of the striking ball remained the same in all the experiments. b was measured depending on number of balls in the chain. The value was measured related to center of the last ball in the chain. It's obvious that kinetic energy of the ball being dashed out is proportional to b. α coefficient can be determined from dependence of kinetic energy of the ball being dashed out on number of balls in the chain as follows on Figure 5.

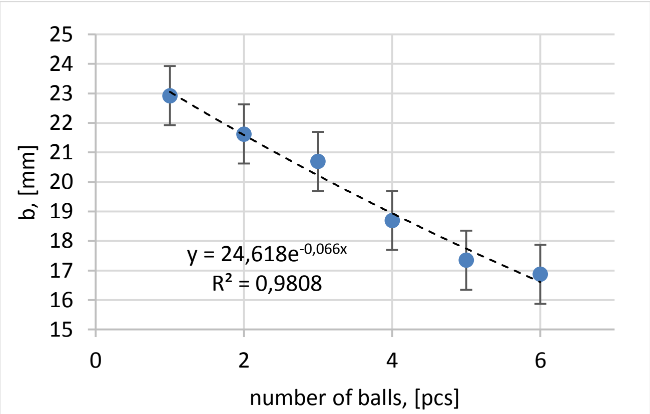

Figure 5. Dependence of *b on number of balls in the chain (n).*

Common α coefficient is determined as follows: $\alpha = e^{-k}$, where k is an exponential factor in the approximation curve.

Building a mathematical modal

Let's transform (7) based on the assumption that the initial speed of the first ball equals to zero, the last ball doesn't rotate, and taking into account the specificity of the energy dissipation:

$$\frac{mv_2^2}{2} = \frac{A_1}{1.4}\alpha^n - A_2(n) \quad (8)$$

The final speed of the ejected ball being dashed out can be expressed as follows:

$$v_2 = \sqrt{\frac{2}{m}\left(\frac{\int_{-\infty}^{-r-c} F_1(x,n)dx}{1.4}\alpha^n - \int_{2r\left(n-\frac{1}{2}\right)+c}^{+\infty} F_2(x,n)dx\right)} \quad (9)$$

Discussion of the results

The position of the magnet is determined by the number of balls before and after it. The more balls there are before the magnet, the worse the result will be, so in order to obtain the best result, the magnet should be placed first. This can be proved by the following results obtained during the experiments (Figure 6).

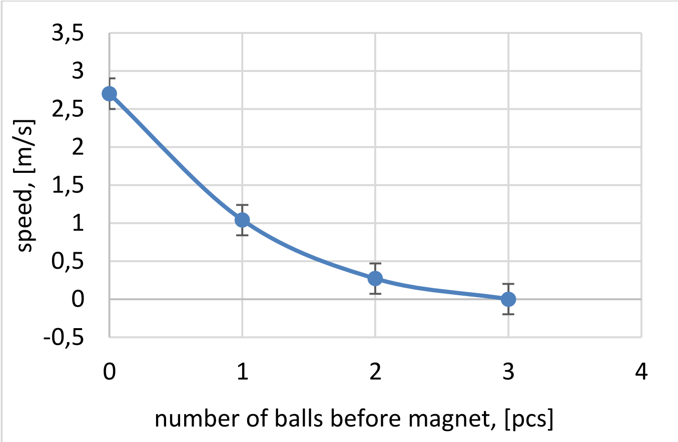

Figure 6. The speed of the ejected ball depending on the number of balls before the magnet. The diameter of balls is 8.6 mm. Number of balls after magnet is 4.

Hence, in this paper we are focusing only on the results obtained when changing the number of balls after the magnet.

We should note that the system does not always allow to change just one of its parameters, with all the others remaining the same. The configuration of system makes strong effect on all parameters. For that reason we chose the only magnet. It was used in all experiments because of its size and constructional features of the setup. It has 8x8x4 mm size, made of NdFeB alloy. When the magnet is a lot larger than the balls (2 or more times), the balls in the system do not remain stable. In this case holding force of magnet won't allow the ball to be ejected with a high speed. Such a system was, therefore, not optimal for our study.

In theory the choice of the optimal size of the ball occurs for the following reasons. Obviously, with the increasing of the radius of the ball its mass is growing considerably faster than the force of the magnet affecting the ball (Figure 7).

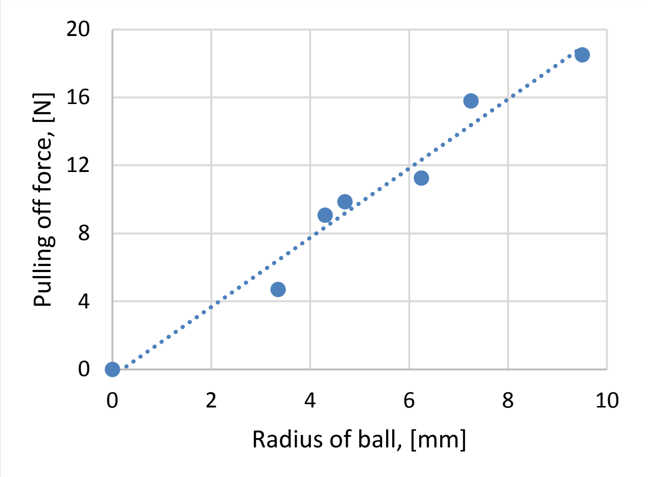

Figure 7. Experimental dependence $F(r_{ball})$.

A simple evaluation shows that velocity of ejected ball should be inversely proportional to the ball radius. It is proved by the experimental results (Figure 8).

$$E = \frac{mv^2}{2} \sim Fl$$

$$F \sim R\,(Figure \cdot 7); \quad m \sim R^3$$

$$v^2 \sim \frac{1}{R^2};$$

$$v \sim \frac{1}{R}(Figure \cdot 8)$$

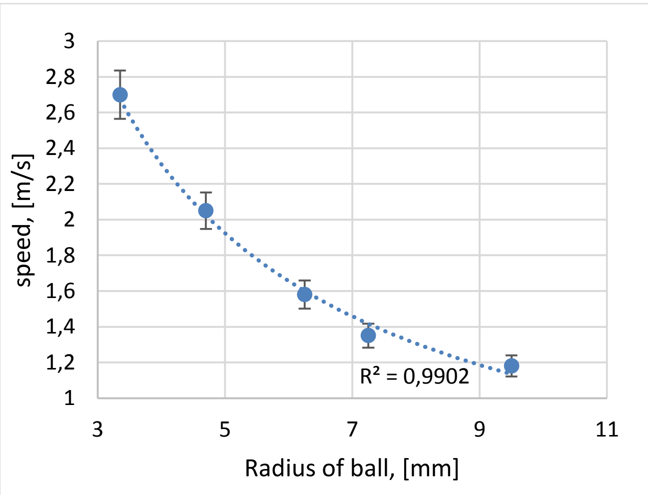

Figure 8. Dependence of velocity on balls radius.

Basing on this data we conclude that the best balls are the smallest ones ($r = 3{,}5$ mm).

For each type of balls we observe the maximum speed corresponding to a certain number of balls after magnet. It can be described as follows: the optimal amount of balls will correspond to minimal energy loss. Also, it's necessary to consider dependences F_1 on number of balls after magnet (Figure 9).

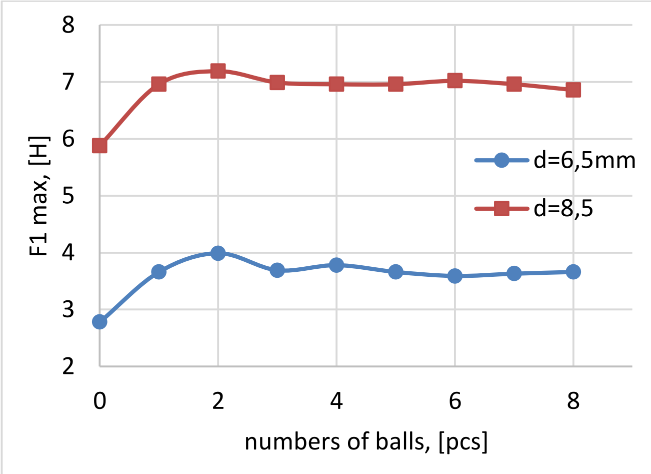

Figure 9. Dependence F_1 on number of balls.

It's not reasonable to determine intrinsic magnetic moment of the balls to conduct the theoretical calculations. Experimental values of the pull-off force $F_1(x,n)$ are an integral characteristic that allows to consider all the factors. Hence, in our calculations we were using $F_1(x,n)$ experimental data.

We get final results after substituting all data into formulae (9). On Figure 10 we can see the comparison of theoretical and experimental results for three smallest available balls. As we can see, our theoretical model describes system well.

Also, we made a series of additional experiments, showing that this phenomenon isn't connected with presence of magnetic field and has only mechanical nature. In the first series we reduced magnet and despite that the phenomenon of ejection of several balls was observed. In second series we changed balls of steel to plastic balls and the phenomenon still was observed (link: watch?=ATz_IQHRZWI).

To avoid this appearance we made several experiments with strongly fixed line of balls (except the last one). It gave the growth of velocity by 10% on average.

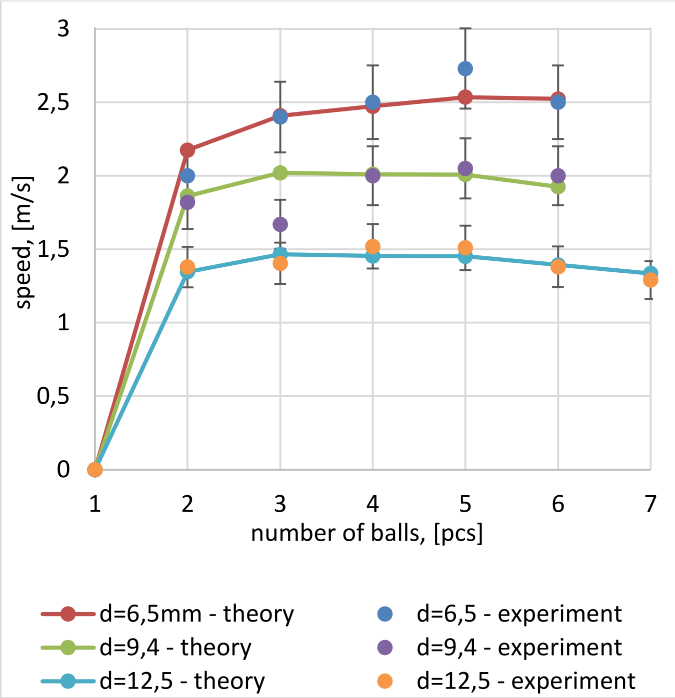

Figure 10. Results of experiment. Dependence v(n) for different numbers of balls for different ball sizes.

Dependence of the maximum speed on ball radius.

To optimize setup and on order to follow the task we made experiments with several stages (see video from the condition). On Figure 11 we can see experimental dependence of the final velocity on the number of stages, and a comparison with values got by the theory.

For convenience, we used setup with big balls (8,5 mm diameter) which gains only 1,8 m/s maximal speed on one staged setup.

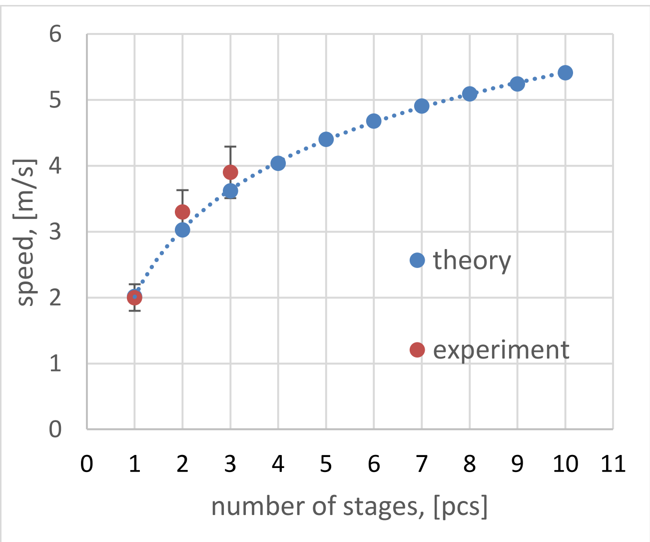

Figure 11. Results of experiment. Dependence v(n) for different stage numbers.

Final velocity increases with the increase of number of stages as long as the energy losses in the system are equal to the afflux of energy.

Conclusions

Finally, in this study we found dependence of the speed on the position of the magnet in the line. There should be no balls between the magnet and the striking ball. There is the optimal number of balls after magnet for each system. It varies from 2 to 5 and is determined by geometrical parameters of magnet, balls material, dissipation in system and the quantity of seed. Then we defined the parameters of the optimal system and used the method of optimization for getting the maximal speed - using several consecutive stages. With the increasing of the number of stages, final speed increases logarithmically, which is proved by the theory by substituting into formulae (9) initial speed, which isn't equal to 0 m/s. On higher speeds we should consider the air resistance, decrease of the recovery factor and other factors, which is beyond this model. We got the 2,7 m/s maximal speed for one staged setup. For 3 stages maximal speed was 3,8 m/s.

References

1. G. S. Landsberg (2000). Textbook of Elementary Physics: Volume 1: Mechanics Heat Molecular Physics. ISBN13: 978-0-8987-5036-2.

2. L. D. Landau, E.M. Lifshitz (1976). Mechanics. Vol. 1 (3rd ed.). ISBN 978-0-7506-2896-9.

Links on YouTube

1. YouTube Channel: youtube.com/UCqxGTfguulhfGid0MABb-eA
2. youtube.com/watch?v=4kyTOaJEQok
3. youtube.com/watch?v=PdyR-ZadS2c
4. youtube.com/watch?v=eARsFW4yY5A